UCD–95–16

Exact and Approximate Radiation Amplitude Zeros — Phenomenological Aspects

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Abstract

We review the phenomenological aspects of the exact and approximate Radiation Amplitude Zeros (RAZ) and discuss the prospects of searches for these zeros at current and future collider experiments.

I. EXACT AND APPROXIMATE RADIATION AMPLITUDE ZEROS

More than 15 years ago, the pioneer studies on vector-boson pair production [1–3] revealed a surprise: the angular distribution for $f_1\bar{f}_2 \to W^-\gamma$ develops a pronounced zero [3,2] at

$$\cos\theta = (Q_{f_1} + Q_{f_2})/(Q_{f_1} - Q_{f_2}), \tag{1}$$

where θ is the W^- scattering angle with respect to the incident fermion (f_1) direction in the center of mass (c.m.) frame, and Q_{f_i} the electric charge of fermion f_i. Figure 1 demonstrates this unusual angular distribution for $e^-\nu$, $d\bar{u} \to W^-\gamma$ processes, in which the zero occurs at $\cos\theta = 1, -1/3$, respectively. The authors of Ref. [3] stated in the abstract that "... We can offer no explanation for this behavior".

In fact, it is not difficult to see what is happening for some simple cases. Take $d\bar{u} \to W^-\gamma$ as an example. There are three Feynman diagrams to contribute at the Born level: a t-channel diagram with an amplitude proportional to Q_u/t, a u-channel diagram proportional to Q_d/u, and an s-channel diagram proportional to $Q_{W^-}/(s-M_W^2)$, where $t=(p_d-p_W)^2=-\frac{1}{2}(s-M_W^2)(1-\cos\theta)$. Notice the charge relation in the Standard Model $Q_d - Q_u = Q_{W^-}$, and the kinematical relation $s-M_W^2=-t-u$, one can easily cast the amplitude into the form

$$\mathcal{M} \sim (\frac{Q_u}{t}+\frac{Q_d}{u})F(\sigma_i,\lambda_i,p_i), \tag{2}$$

where $F(\sigma_i,\lambda_i,p_i)$ denotes a reduced matrix element as a function of the fermion helicity σ_i, vector-boson polarization λ_i and the external momenta p_i. We see immediately that this amplitude develops a zero at a special angle determined by Eq. 1.

Not long after this discovery, several groups [4–6] further examined this interesting feature. It was found that in gauge theories, any tree-level 4-particle (spin ≤ 1) Feynman amplitudes with one or more massless gauge particles can be factorized into two factors, one of which contains the dependence of internal quantum numbers (such as charges) and the other contains the dependence of spin and polarization indices [4]. This factorization is a special case for a more general theorem [5], which states that *for a tree-level n-particle (spin ≤ 1) amplitude with one photon, the amplitude develops a zero when the factor $Q_i/p_i\cdot q$ are equal for $i=1,2...n-1$, where Q_i and p_i are the charge and momentum for the i^{th} particle, respectively, and q the photon momentum.* There is certainly a deeper explanation for this phenomena, having something to do with the relationship between the internal gauge symmetry and the space-time symmetry. One can find a very nice discussion in Bob Brown's talk at this conference [7], or from the classical papers on this subject [5]. Following the literature, we will call those zeros Radiation Amplitude Zeros (RAZ) [8].

It should be noted, however, that

- not all of the RAZ occur in physical region — in fact, most of them do not. The above theorem can be translated into an intuitive necessary condition for RAZ to occur in

physical region: *along with a massless gauge boson, the other particles involved in the process must have the same sign of electric charges.* We will call this condition the "same-sign rule".

- although loop diagrams (and bubbles) do not significantly alter the nature of RAZ [9], higher order real emissions spoil the RAZ [10–12]. It was suggested [11] that one can regain the Born-level kinematics by vetoing additional final state particles, thus recovering an "approximate" zero in practice.

It is natural to ask what may happen in a theory with a spontaneously broken gauge symmetry, such as the Standard Model (SM). It is conceivable that the radiation of a Z-boson may have some similarity to that of a photon. For the case of $d\bar{u} \to W^- Z$, the amplitude can be written as [13]

$$\mathcal{M} \sim X F_X(\sigma_i, \lambda_i, p_i) + Y F_Y(\sigma_i, \lambda_i, p_i), \tag{3}$$

where X and Y are combinations of coupling factors

$$X = \frac{s}{2}\left(\frac{g_-^{f_1}}{u} + \frac{g_-^{f_2}}{t}\right), \quad Y = g_-^{f_1}\,\frac{M_Z^2\, s}{2\,u\,(s - M_W^2)}\,, \tag{4}$$

with the left-handed neutral current couplings $g_-^{f_1} - g_-^{f_2} = Q_W \cot\theta_\mathrm{w}$, and $F_{X,Y}(\sigma_i, \lambda_i, p_i)$ contain the spin dependent part and is roughly proportional to the product of the vector-boson wave functions $\epsilon_\mathrm{w}^* \cdot \epsilon_\mathrm{z}^*$. It is obvious that without the Y-term, the helicity amplitudes would factorize. In this case, all amplitudes would simultaneously vanish for $g_-^{f_1}/u + g_-^{f_2}/t = 0$, analogous to the $W\gamma$ case in Eq. 2. Since Y is directly proportional to M_Z^2, one may naively expect full factorization when $M_Z^2 \ll s$. In fact, in the high energy limit, only three helicity amplitudes remain non-zero:

$$\begin{aligned}
\mathcal{M}(\lambda_\mathrm{w} = \pm, \lambda_\mathrm{z} = \mp) &\longrightarrow \frac{1}{\sin\theta}\,(\lambda_\mathrm{w} - \cos\theta)\left[(g_-^{f_1} - g_-^{f_2})\cos\theta - (g_-^{f_1} + g_-^{f_2})\right], \\
\mathcal{M}(\lambda_\mathrm{w} = 0, \lambda_\mathrm{z} = 0) &\longrightarrow \frac{1}{2}\,\sin\theta\,\frac{M_Z}{M_W}\,(g_-^{f_2} - g_-^{f_1})\,.
\end{aligned} \tag{5}$$

While the dominant amplitudes $\mathcal{M}(\pm, \mp)$ fully factorize in the high energy limit, $\mathcal{M}(0,0)$ behaves differently. This can be traced to the special energy-dependence of the polarization

vectors for longitudinal vector bosons, $\epsilon_{\rm v} \sim \sqrt{s}/M_V$. Since the Y-term in Eq. 3 goes like $(M_Z^2/s)\,\epsilon_{\rm w}^* \cdot \epsilon_{\rm z}^*$, the $\mathcal{M}(0,0)$ amplitude remains finite at high energies.

The combined effect of the zero in $\mathcal{M}(\pm,\mp)$ and the relatively small contributions from the remaining helicity amplitudes results in an approximate zero for the $f_1\bar{f}_2 \to W^\pm Z$ differential cross section at

$$\cos\theta \simeq (g_-^{f_1} + g_-^{f_2})/(g_-^{f_1} - g_-^{f_2}) \simeq \begin{cases} \ \ \frac{1}{3}\tan^2\theta_{\rm w} \simeq 0.1 \quad \text{for } d\bar{u} \to W^- Z\,, \\ -\tan^2\theta_{\rm w} \simeq -0.3 \ \ \text{for } e^-\bar{\nu}_e \to W^- Z\,. \end{cases}$$

This is illustrated in Fig. 2 where the differential cross sections are shown for $e^-\bar{\nu}_e \to W^- Z$ and $d\bar{u} \to W^- Z$ for $(\lambda_{\rm w}, \lambda_{\rm z}) = (\pm,\mp)$ and $(0,0)$, as well as the unpolarized cross section, which is obtained by summing over all W- and Z-boson helicity combinations (solid line). For both reactions, the total differential cross section displays a pronounced minimum at the location of the zero in $\mathcal{M}(\pm,\mp)$. Due to the $1/\sin\theta$ behaviour of $\mathcal{M}(\pm,\mp)$, the $(+,-)$ and $(-,+)$ amplitudes dominate outside of the region of the zero. In order to demonstrate the influence of the zero in $\mathcal{M}(\pm,\mp)$ on the total angular differential cross section, the $\cos\theta$ distribution for $e^+e^- \to ZZ$ has been included in Fig. 2a). The zero in the $(\pm,\mp)$ amplitudes causes the minimum in the WZ case to be much more pronounced than the minimum in $e^+e^- \to ZZ$.

It is important to note that the RAZ are the direct results from subtle gauge cancellation. Non-standard couplings, such as those Δg_1, $\Delta\kappa$ and λ [14] spoil these cancellations and eliminate the (approximate) zeros. This can be seen from the additional contributions to the SM amplitudes, for the $W\gamma$ process,

$$\Delta\mathcal{M}_{\rm w\gamma}(\pm,\pm) = \frac{F}{2}\,\sin\theta\left[\Delta\kappa + \frac{\lambda}{r_{\rm w}}\right], \tag{6}$$

$$\Delta\mathcal{M}_{\rm w\gamma}(0,\pm) = \frac{F}{2}\,\frac{(1+\lambda_\gamma\cos\theta)}{\sqrt{2r_{\rm w}}}\left[\Delta\kappa + \lambda\right], \tag{7}$$

where $F = V_{f_1 f_2} e^2/\sqrt{2}\sin\theta_{\rm w}$ and $r_{\rm v} = M_V^2/s$; the corresponding contributions to the WZ production amplitudes are

$$\Delta\mathcal{M}_{\rm wz}(\pm,\pm) = \frac{F}{2}\,\frac{Q_W\,\cot\theta_{\rm w}}{1-r_{\rm w}}\,\beta\sin\theta\left[\Delta g_1 + \Delta\kappa + \frac{\lambda}{r_{\rm w}}\right], \tag{8}$$

$$\Delta\mathcal{M}_{\rm wz}(0,0) = \frac{F}{2}\,\frac{Q_W\ \cot\theta_{\rm w}}{1-r_{\rm w}}\,\frac{\beta\sin\theta}{\sqrt{2r_{\rm w}2r_{\rm z}}}\,2\left[\Delta g_1(1+r_{\rm w}) + \Delta\kappa\, r_{\rm z}\right], \tag{9}$$

$$\Delta\mathcal{M}_{\rm wz}(\pm,0) = \frac{F}{2}\,\frac{Q_W\ \cot\theta_{\rm w}}{1-r_{\rm w}}\,\frac{\beta(1-\lambda_{\rm w}\cos\theta)}{\sqrt{2r_{\rm z}}}\left[2\,\Delta g_1 + \lambda\,\frac{r_{\rm z}}{r_{\rm w}}\right], \tag{10}$$

$$\Delta\mathcal{M}_{\rm wz}(0,\pm) = \frac{F}{2}\,\frac{Q_W\ \cot\theta_{\rm w}}{1-r_{\rm w}}\,\frac{\beta(1+\lambda_{\rm z}\cos\theta)}{\sqrt{2r_{\rm w}}}\left[\Delta g_1 + \Delta\kappa + \lambda\right], \tag{11}$$

where $\beta = [(1-r_{\rm w}-r_{\rm z})^2 - 4r_{\rm w}r_{\rm z}]^{1/2}$. Due to angular momentum conservation, the $(\pm,\mp)$ amplitudes which dominate in the SM do not receive any contributions from the anomalous couplings. The amplitude zeros in these two helicity configurations for both $W\gamma$ and WZ channels thus remain exact. All other helicity amplitudes are modified in the presence of non-standard $WW\gamma/WWZ$ couplings. At high energies the anomalous contributions grow proportional to $\sqrt{s}$ (s) for $\Delta\kappa$ (Δg_1 and λ) and eventually dominate the cross section. The nature of the RAZ is thus sensitive to new physics in the vector-boson sector.

II. PROSPECTS OF EXPERIMENTAL SEARCHES FOR RAZ

Clearly, the radiation amplitude zeros (RAZ) are a very interesting feature of gauge theories and it would be desirable to experimentally observe this distinctive phenomena. However, we emphasize that studying these "zeros" is not to search for "nothing". Rather, we would hope to find new physics in the vector-boson sector and the amplitudes near RAZ are especially sensitive to the deviation from the SM. This is the motivation to examine the feasibility of experimental searches for RAZ.

A. $W\gamma$ Production at Hadron Colliders: $p\bar{p}, pp \to W^\pm\gamma \to l^\pm\nu\gamma$

The successful $p\bar{p}$ collider experiments at the Fermilab Tevatron may provide suitable environment for searching for RAZ and for testing the anomalous gauge boson couplings [15]. However, it is non-trivial to carry out the searches for the RAZ experimentally. The problems, both theoretical and experimental, include:

1. *reconstruction of the $q\bar{q}$ c.m. frame:* it is impossible to non-ambiguously reconstruct the parton c.m. frame to define the scattering angle to obtain $d\sigma/d\cos\theta$ since the reconstruction of the neutrino momentum (p_ν) from constraint $(p_l + p_\nu)^2 = M_W^2$ is subject to a two-fold ambiguity [16,17].

2. *z-axis along the incident fermion moving direction:* in hadron colliders, there are two types of parton-level contributions to the same final state: $d_1\bar{u}_2 \to W^-\gamma$ and $\bar{u}_1 d_2 \to W^-\gamma$. Since the polar angle θ is defined with respect to incident fermion moving direction $\vec{p}_d$, it is then impossible to non-ambiguously identify the direction of z-axis (along the d-quark). In $p\bar{p}$ collisions at Tevatron energies, due to the valence quark dominance, the contribution from $d_1\bar{u}_2$ is much larger than that from $\bar{u}_1 d_2$, so that one can simply assign the z-axis along the proton direction. However, in pp collisions, those contributions are equal, making the z-axis identification intrinsically impossible.

3. *higher order corrections:* the RAZ in $d\bar{u} \to W^-\gamma$ is exact only for the $2 \to 2$ Born-level process. Additional jets from higher order QCD radiation [10–12] will spoil the subtle cancellation and thus fill up the zero. One has to reject (or veto) the additional jets to recover the Born-level kinematics [11].

4. *W^- radiative decay:* for the channel $d\bar{u} \to W^-\gamma \to e^-\bar{\nu}_e\gamma$, a single W^- (Drell-Yan) production with subsequent radiative decay $d\bar{u} \to W^- \to e^-\bar{\nu}_e\gamma$ gives the same final state but different kinematical structure. Those events should be kept separated. This could be achieved by imposing a transverse mass cut [17–19] slightly above M_W, $M_T(l^\pm\nu, \gamma) > 90$ GeV.

5. *backgrounds:* the most severe background for the $W^-\gamma$ final state seems to be the misidentification of a photon from a jet $j \to \gamma$, due to the much larger production rate for W^-j. Good γ-j discrimination factor is needed to successfully identify the signal.

The first attempt to realistically study the RAZ at the Tevatron was carried out in Ref. [17]. Due to the two-fold ambiguity in constructing the neutrino momentum, the authors studied two polar angle distributions $\cos\theta^*_+$ and $\cos\theta^*_-$, corresponding to the two solutions for $\cos\theta^*_\nu > \cos\theta^*_e$ and $\cos\theta^*_\nu < \cos\theta^*_e$, respectively. Although one is unable to tell the correct p_ν solution on an even-by-event basis, it is seen from Fig. 3 that $\cos\theta^*_-$ reflects the zero location better. This can be understood in terms of the $V-A$ coupling. Namely, e^- ($\bar\nu_e$) prefers to move in the forward (backward) direction so that $\cos\theta^*_\nu < \cos\theta^*_e$, which corresponds to the $\cos\theta^*_-$ solution. Fig. 3 also demonstrates the anomalous coupling effects that tend to fill up the dip.

The RAZ in Eq. 1 corresponds to the photon rapidity in c.m. frame

$$y^*_\gamma = \frac{1}{2}\ln\frac{1+\cos\theta_\gamma}{1-\cos\theta_\gamma} = \frac{1}{2}\ln(-\frac{Q_2}{Q_1}), \tag{12}$$

which gives $y^*_\gamma \simeq \pm 0.35$ for $W^\mp\gamma$ channel. As a direct reflection of the RAZ, the photon rapidity spectrum in the c.m. frame develops a clear dip in the central region after summing over the two solutions for p_ν [18,19]. A problem arises when we include QCD radiative corrections [12]. Although moderate at the Tevatron energies, the QCD corrections tend to fill up the dip and to increase the cross section in a similar way as the anomalous couplings [11]. Figure 4 shows the differential cross section for the photon rapidity in the reconstructed center of mass frame for the reaction $p\bar p \to W^+\gamma \to e^+\nu_e\gamma$ at $\sqrt{s} = 1.8$ TeV in the SM. The inclusive next-to-leading-order (NLO) differential cross section [solid line in a)] is seen to be significantly larger than the Born-level leading-order (LO) approximation [dot-dashed line in b)] and tend to fill in the dip near zero. However, the NLO $W\gamma + 0$ jet exclusive differential cross section (dotted line) is comparable to the Born-level LO result. This important observation implies that if we study the 0-jet exclusive process $p\bar p \to W\gamma +$ 0-jet $\to e\nu_e +$ 0-jet, namely, if we veto the extra jet(s) from higher order QCD processes, we recover most of the feature in the Born level and thus regain the sensitivity to study the anomalous couplings.

It is noted that the RAZ for $p\bar{p} \to W^{\pm}\gamma \to e^{\pm}\nu_e$ occur in the central region $\cos\theta = \pm\frac{1}{3}$ (and $y^*_\gamma \simeq 0$ averagely). Therefore, deviations from the SM will largely happen in high transverse momentum $p_T(\gamma)$ region. This feature has been carefully examined in a recent paper [20]. It is shown that, as a function of a cutoff on the photon transverse momenta $p_T^{min}(\gamma)$, the ratio of integrated cross sections $R_{\gamma,l} = \sigma(\gamma Z)/\sigma(\gamma W)$ for $Z\gamma$ process (which has no RAZ) and for $W\gamma$ process (which has a RAZ) is a clear indication of a zero behavior, as shown in Fig. 5. We see that in high $p_T(\gamma)$ region, the rate for $W\gamma$ process is significantly smaller than that of $Z\gamma$ process. In contrast, the ratio versus a cutoff on a jet transverse momentum $p_T^{min}(j)$ for Zj and Wj production is flat over a large $p_T(j)$ range. The advantage of looking at the cross section ratio versus $p_T^{min}(\gamma)$ is to have avoided the c.m. frame and z-axis ambiguities, while the price to pay is to lose the information about the exact RAZ location.

Some more interesting observation has been made recently in Ref. [21]. Recall the rapidity in the lab frame (y) is a sum of that in c.m. frame (y^*) and a term reflecting the c.m. frame motion

$$y = y^* + \frac{1}{2}\ln\left(\frac{x_1}{x_2}\right), \tag{13}$$

where $x_{1,2}$ are the parton momentum fractions.

If we take the rapidity difference between the photon and the W, then the difference is invariant under the longitudinal boost. Therefore, the rapidity correlation between W-γ in the c.m. frame is preserved in the lab frame

$$\Delta y = y_\gamma - y_W = y^*_\gamma - y^*_W \simeq -0.4. \tag{14}$$

We thus have a chance to avoid the frame ambiguity if we choose the variable in such a clever way. Fig. 6 demonstrates the rapidity correlation in the lab frame [21]. We see an impressive "valley" for the rapidity correlation, given by $y_\gamma - y_W \simeq -0.4$. In order to implement this idea more realistically, we must use the final state momentum of $l^{\pm}$, rather than that of W. Fortunately, based on helicity arguments, the charged lepton in $W\gamma$ process goes dominantly along the W moving direction, so that W-γ rapidity correlation is largely preserved,

$$\Delta\eta(\gamma, l) = \eta(\gamma) - \eta(l) \simeq -0.3. \tag{15}$$

One could directly study the rapidity difference, which would have the advantage for higher statistics than the double differential cross section. This is shown in Fig. 7. The curve for the pseudorapidity difference in the SM (solid) presents a clear dip at -0.3. The authors of Ref. [21] have also estimated the error bars for expected statistical uncertainties with an integrated luminosity of 22 pb^{-1}. Since the CDF/D0 collaborations have accumulated about 100 pb^{-1} each (at the time of writing), one can anticipate that an experimental study along this line may first observe the clear dip reflecting the RAZ. The effects from anomalous couplings are also demonstrated in the figure.

Finally, two remarks are in order. First, we have thus far concentrated on Tevatron energies. At the LHC, due to the more severe problems regarding the z-axis definition and much larger QCD corrections to the Born amplitudes, the conclusions in studying the RAZ seem rather pessimistic. Secondly, we have not discussed much about the background issue. It turns out that if we could achieve a $j \to \gamma$ misidentification factor at a level of 10^{-3}, the background may not be too severe [21,15].

B. W Radiative Decay: $W^\pm \to l^\pm \nu\gamma$, $q\bar{q}'\gamma$

It was shown [22] that the W-radiative decay, $W \to f_1 \bar{f}_2 \gamma$, also presents a RAZ. Refs. [17–19] studied the process $p\bar{p} \to W^\pm \to e^\pm \nu\gamma$ at the Tevatron. This process develops a zero at the kinematical boundary $\cos\theta_{l\gamma} = -1$ in W-rest frame. It can be effectively separated from the $W\gamma$ associated production by imposing a transverse mass cut, $M_T(l^\pm\nu, \gamma) < 90$ GeV; and it also has larger statistics. However, the RAZ is less pronounced due to the single-zero behavior [17,22] and the difficulty for W-rest frame reconstruction. It is therefore less sensitive to anomalous couplings.

Ref. [23] discussed the zero in the hadronic decay process $W^- \to d\bar{u}\gamma$. In this case, it is a double-zero as usual and the W-rest frame reconstruction may be relatively easier. However, the event identification may be difficult in hadron collider experiments; and it will

suffer from low statistics for $e^+e^- \to W^+W^-$ with a radiative W decay.

C. WZ Production: $p\bar{p}, pp \to W^\pm Z \to l^\pm \nu l^+ l^-$

As discussed earlier, the Born amplitude for $q_1\bar{q}_2 \to W^\pm Z$ develops a zero at high energies [13] at $\cos\theta \simeq \pm\frac{1}{3}\tan^2\theta_{\rm W} \approx \pm 0.1$. Following the proposal of studying the rapidity correlation analogous to $\Delta\eta(\gamma,\ell)$ for $p\bar{p} \to W^+\gamma$ process in Fig. 7, one can examine the rapidity correlation [24] via $\Delta y(Z,\ell_1) = y(Z) - y(l_1)$ where l_1 is the charged lepton from W decay. Figure 8 shows the differential cross section $d\sigma/\Delta y$. There is a dip near 0.1 as predicted in the SM (solid curve), although it will not be easy to convincingly establish the effect due to the less pronounced dip for this channel and limited number of $W^\pm Z \to \ell_1^\pm \nu_1 \ell_2^+ \ell_2^-$ events expected (see the estimated statistical error bars in the figure for 10 fb^{-1} luminosity). At the LHC energies, the zero is further washed out due to larger QCD radiative corrections and the z-axis ambiguity.

It is amusing to note [13] that $e\nu_e$ or $\mu\nu_\mu$ collisions above the WZ threshold would in principle provide a clean environment for event reconstruction. The location of the zero at $\cos\theta \approx \pm 0.3$ is ideal for experimental studies of the $W^\pm Z$ final state, unlike the case for $e^-\bar{\nu}_e \to W^-\gamma$ where the zero is located at the kinematical boundary ($\cos\theta = 1$) resulting a single-zero.

D. $qq' \to qq'\gamma$ And $eq \to eq\gamma$

Certain single photon radiation processes in quark scattering, such as

$$uu \to uu\gamma, \quad u\bar{d} \to u\bar{d}\gamma, \quad dd \to dd\gamma, \quad d\bar{u} \to d\bar{u}\gamma \tag{16}$$

present a RAZ [25] at

$$\cos\theta_\gamma = (Q_2 - Q_1)/(Q_2 + Q_1), \tag{17}$$

where Q_1 and Q_2 are the electric charges for initial state quarks and θ_γ the photon scattering angle with respect to the incident quark. But some other processes such as

$$u\bar{u} \to u\bar{u}\gamma, \quad ud \to ud\gamma, \quad d\bar{d} \to d\bar{d}\gamma, \quad du \to du\gamma \tag{18}$$

do not. The cause for the difference is the "same-sign rule", as stated earlier. The locations of the zeros are clearly sensitive to the fractional charges of the quarks [25], although there are no triple vector-boson self-interactions involved. However, after convoluting with the hadron structure functions, the RAZ becomes a dip. At low energies where the valence quarks dominate, there is a good chance one could find the RAZ in these processes. Our experimental colleagues may consider to re-examine the low energy data, such as that at CERN ISR pp collider ($\sqrt{s} \sim 30$ - 60 GeV), for this purpose. At higher energies, such as at the Fermilab Tevatron [26], the QCD multiple-jet processes would completely swamp the RAZ signal.

It is straightforward to calculate the processes $e^{\pm}p \to e^{\pm}X\gamma$ [27,28] by simply replacing one of the quarks by $e^{\pm}$ in processes 16,18. Once again, at low energies where the valence quarks dominate, it is possible to examine the dip resulted from the RAZ. At HERA energies, however, the RAZ effects in $e^{-}p$ collisions seem to be largely washed out, while it is claimed to be more promising in $e^{+}p$ collisions [28], again due to the argument of the "same-sign rule". Inclusion of more realistic experimental simulation may further worsen this situation.

III. RAZ IN THEORIES BEYOND THE STANDARD MODEL

The RAZ is a general feature in gauge theories. There are in fact many more processes beyond the SM in which the RAZ occur. The RAZ theorem has been generalized to supersymmetric theories with massless gaugino emission [29] and RAZ have been found in the exact supersymmetric limit for processes [30] such as

$$d\bar{u} \to \tilde{W}\tilde{\gamma}, \quad \gamma e \to \tilde{W}\tilde{\nu_L} \quad etc.. \tag{19}$$

In this limit, the RAZ locate at the same places as those for the SM partners.

The RAZ is also found in charged Higgs boson production $p\bar{p} \to H^{\pm}\gamma$ [31], although the small Yukawa coupling of $H^{\pm}$ to light fermions would make this process unobservable. A

more promising process is for the decay $H^+ \to t\bar{b}\gamma$ [32] if kinematically accessible. Similarly, the RAZ effects in radiative decays of other charged scalar particles such as lepto-quarks are also studied [33].

IV. SUMMARY

Certain tree-level processes involving massless gauge bosons and charged particles present radiation amplitude zeros (RAZ). With higher order radiative corrections and in a more realistic experimental environment, those zeros are always approximate or become dips. In the SM with a spontaneously broken gauge symmetry, WZ final state develops approximate zeros at high energies. In general, the nature of those zeros is sensitive to the gauge couplings of vector bosons and that of fermions as well. Studying these RAZ experimentally may thus provide probes to physics beyond the SM.

Progress has been made in studying the RAZ both theoretically and experimentally, *e. g.* $p\bar{p} \to W^\pm\gamma \to l^\pm\nu\gamma$ and $W^\pm Z \to l^\pm\nu l^+ l^-$ at Fermilab Tevatron energies. Other processes such as $qq' \to qq'\gamma$ at low energies, and $e^+p \to e^+X\gamma$ at HERA should be examined at a level with realistic experimental acceptance to draw further conclusion. It is clearly challenging to experimentally observe those "approximate" zeros. Hopefully one day, we would be able not only to observe the RAZ, but also in so doing to find some hints on new physics in the vector-boson sector.

ACKNOWLEDGEMENT

I would like to thank Uli Baur, Steve Errede and Thomas Müller for their invitation and their nice organization of such a stimulating conference. I am also grateful to Bob Brown for illuminating discussions during the conference. This work was supported in part by the U.S. Department of Energy under Contract No. DE-FG03-91ER40674.

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FIGURES

FIG. 1. Differential cross sections $d\sigma(\lambda_{\rm w}, \lambda_\gamma)/d\cos\theta$ for a). $e^-\bar{\nu}_e \to W^-\gamma$ and b). $d\bar{u} \to W^-\gamma$, where θ is the polar angle between W^- and the incident fermion (e^- or d) in the c.m. frame. For comparison, the differential cross section for $e^+e^- \to ZZ$, in which there is no RAZ, has been included in a).

FIG. 2. Differential cross sections $d\sigma(\lambda_{\rm w}, \lambda_{\rm z})/d\cos\theta$ for a). $e^-\bar{\nu}_e \to W^-Z$ and b). $d\bar{u} \to W^-Z$, where θ is the polar angle between W^- and the incident fermion (e^- or d) in the c.m. frame. For comparison, the differential cross section for $e^+e^- \to ZZ$, in which there is no RAZ, has been included in a).

FIG. 3. Differential cross sections a). $d\sigma/d\cos\theta^*_+$ b). $d\sigma/d\cos\theta^*_-$ for $p\bar{p} \to W^-\gamma$, $W^- \to e^-\bar{\nu}_e$. Note that θ^* here is the polar angle between γ and p in the c.m. frame. Effects from an anomalous coupling κ are also shown, where $\kappa = 1$ corresponds to the SM results. Acceptance cuts are described in Ref. [17].

FIG. 4. The differential cross section for the photon rapidity in the reconstructed center of mass frame for the reaction $p\bar{p} \to W^+\gamma \to e^+\nu_e\gamma$ at $\sqrt{s} = 1.8$ TeV in the SM. a) The inclusive NLO differential cross section (solid line) is shown, together with the $\mathcal{O}(\alpha_s)$ 0-jet (dotted line) and the (LO) 1-jet (dashed line) exclusive differential cross sections. b) The NLO $W\gamma + 0$ jet exclusive differential cross section (dotted line) is compared with the Born-level LO differential cross section (dot-dashed line). A jet is defined as $p_T^j > 10$ GeV and $|\eta^j| < 2.5$. Other cuts imposed are described in Ref. [11].

FIG. 5. The ratio of integrated cross sections as a function of the minimum transverse momentum of the photon, $p_T^{min}(\gamma)$, at the Tevatron. The dashed line shows the corresponding ratio of Zj to $W^\pm j$ cross sections for comparison.

FIG. 6. The double differential distribution $d^2\sigma/dy_\gamma dy_W$ for $p\bar{p} \to W^+\gamma \to \ell^+\nu\gamma$, $\ell = e$, μ, in the Born approximation at the Tevatron (1.8 TeV). The cuts imposed are described in Ref. [21].

FIG. 7. The pseudorapidity difference distribution, $d\sigma/d\Delta\eta(\gamma,\ell)$, for $p\bar{p} \to W^+\gamma$, $W^\pm \to \ell^+\nu$ with $\ell = e$, μ, at the Tevatron in the Born approximation for anomalous $WW\gamma$ couplings. The curves are for the SM (solid), $\Delta\kappa_0 = 2.6$ (dashed), and $\lambda_0 = 1.7$ (dotted). Only one coupling is varied at a time. The error bars indicate the expected statistical uncertainties for an integrated luminosity of 22 pb^{-1}. The cuts imposed are described in Ref. [21].

FIG. 8. The differential cross section for the rapidity difference $\Delta y(Z,\ell_1)$ for $p\bar{p} \to W^+Z + X \to \ell_1^+\nu_1\ell_2^+\ell_2^- + X$ at $\sqrt{s} = 1.8$ TeV. The solid and dot-dashed curves show the inclusive NLO and the LO SM prediction, respectively. The dashed and dotted lines give the results for $\Delta\kappa^0 = +1$ and $\Delta\kappa^0 = -1$. The error bars associated with the solid curves indicate the expected statistical uncertainties for an integrated luminosity of 10 fb^{-1}. The cuts imposed are described in Ref. [24].

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